

Side-channel-free quantum key distribution

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Quantum key distribution (QKD) offers the promise of absolutely secure communications. However, proofs of absolute security often assume perfect implementation from theory to experiment. Thus, existing systems may be prone to insidious side-channel attacks that rely on flaws in experimental implementation. Here we replace all real channels with virtual channels in a QKD protocol, making the relevant detectors and settings inside private spaces inaccessible while simultaneously acting as a Hilbert space filter to eliminate side-channel attacks. By using a quantum memory we find that we are able to bound the secret-key rate below by the entanglement-distillation rate computed over the distributed states.

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In 1982 Richard Feynman conjectured the use of quantum systems as a technological platform for solving difficult calculations in physics. Eventually this insight led to the field of quantum information processing. As part of the field's growth, it has partly diverged into the two main application domains: computation and communications, though much fundamental and technical overlap still exists. Interestingly, the key application that has started to mature and is now commercially available is quantum cryptography, or more precisely quantum key distribution (QKD) which has quickly moved from the purely theoretical [1–4] to a practical technology [5–9].

How can we explain the impressive industrial uptake of quantum cryptography and its ultimate aim to take over classical systems? The answer lies in the claim of “absolute security” [10]. Unfortunately, while the idea is very compelling, subtle details in implementation may introduce flaws that could, potentially, be open to attack. Specifically, attacks from so called “side channels” represent one of the most elusive threats in practical quantum cryptography, because a system could be vulnerable to side-channel attacks even if it is unbreakable in theory [11, 12]. In fact, the recent approach of “device-independent QKD” [13] makes important advances in handling imperfect implementations, and can even be made by untrusted parties, but does not directly address all possible side-channel attacks, where, for example, detectors may directly receive external probing aimed at seeding or gleaning their readout.

In principle side-channel attacks affect both classical and quantum cryptography, but could be especially devastating for quantum cryptography, precisely because of the proclaimed absolute security “guarantee”. The threat from such attacks has been demonstrated in both lab and installed field settings [12]. Thus, while practical QKD systems have been fighting a trade-off between distance and key generation rate, they are still facing the fundamental problem of guaranteed security, choosing to rely on theoretical promises of absolute security without having any way of authenticating them in practice.

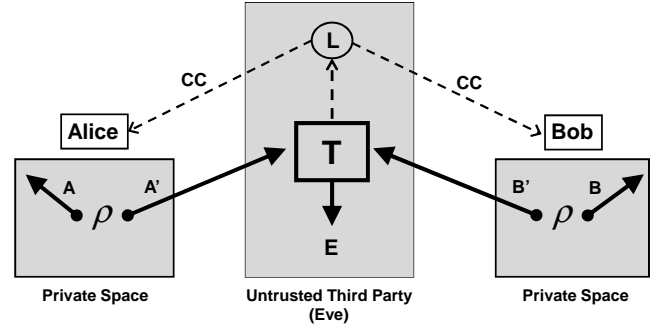


FIG. 1: Private space to private space. The UTP acts as a correlator.

Private spaces: general model

Let us consider the scenario of Fig. 1. Two authenticated parties, Alice and Bob, control two private spaces, \mathcal{A} and \mathcal{B} , respectively. Conventionally, these spaces are assumed completely inaccessible from the outside, i.e., no illegitimate system may enter \mathcal{A} or \mathcal{B} . For this reason every kind of side-channel attack upon the private spaces is assumed excluded. In practice, however, any port can allow a side-channel to enter possibly probing any detector, state-generation or detector settings. To prevent or overcome such attacks, the QKD system must effectively isolate its private spaces: the private space must not be directly involved in either state preparation (for sending) or detection (of incoming states). To overcome such probing side-channel attacks, we propose performing state-generation by collapse of a bipartite entangled state, so that any probe from outside is perfectly isolated from the state-generation “machinery” (see Supplementary Material for an extended discussion). Thus, in a manner akin to teleportation, we replace all real channels with virtual channels. This allows us to physically (and “topologically”) separate all detectors and settings within the private space from external probing, while also acting as a Hilbert space filter [14] against any side channel.

Within its own private space, each party (Alice or

Bob) has a bipartite state ρ which entangles two systems: $\{A, A'\}$ for Alice, and $\{B, B'\}$ for Bob. Systems $\{A, B\}$ are kept within the private spaces, while systems $\{A', B'\}$ are sent to an untrusted third party (UTP), whose task is to perform a quantum measurement and communicate the corresponding result. This untrusted LOCC then allows the creation of correlations between the private systems $\{A, B\}$ that Alice and Bob can exploit to generate a secret-key. In its simplest form an ideal side-channel free QKD scheme reduces to an entanglement swapping setup [15], with the dual teleportation channel acting as an ideal Hilbert space filter. What is unique about our protocol is the ability to completely protect private space settings and detectors from probing side-channel attacks.

In the worst case scenario, the UTP must be identified with Eve herself, whose aim is to eavesdrop the key, or even prevent Alice and Bob from generating the key (i.e., a denial of service). In the most general case, Eve applies a quantum instrument $\mathbf{T} = \{T_l\}_{l=1}^{l_{\max}}$ to the incoming systems $\{A', B'\}$. This is a quantum operation with both classical and quantum outputs. For each classical outcome l , there is a corresponding completely positive (CP) map T_l applied to the systems $\{A', B'\}$ [16]. This means that the global input state $\rho_{AA'} \otimes \rho_{BB'}$ is transformed into the conditional output state

$$\rho_{ABE}(l) \equiv \frac{1}{p(l)}(I_A \otimes I_B \otimes T_l)(\rho_{AA'} \otimes \rho_{BB'}), \quad (1)$$

where E represents an output quantum system in the hands of Eve, while $I_A \otimes I_B$ is the identity channel acting on the private systems $\{A, B\}$. Clearly each outcome l will be found with some probability $p(l)$, depending both on T_l and the input state. As a consequence the classical output of \mathbf{T} can be simply represented by the stochastic variable $L \equiv \{l, p(l)\}$. The quantum output of \mathbf{T} is represented by the system E which is correlated with the private systems $\{A, B\}$ via the conditional state $\rho_{ABE|L}$ specified by Eq. (1). E is the system that Eve will use for eavesdropping. For instance, most generally Eve can store all the output systems E (generated in many independent rounds of the protocol) into a big quantum memory. Then, she can detect the whole memory using an optimal quantum measurement (corresponding to a collective attack).

According to the agreed protocol, the UTP must send a classical communication (CC) to both Alice and Bob in order to “activate” the correlations. Here, Eve has another weapon in her hands, i.e., tampering with the classical outcomes. In order to decrease the correlations between the honest parties, Eve may process the output stochastic variable L via a classical channel

$$p(l'|l) : L \rightarrow L', \quad (2)$$

and then communicate the fake variable $L' = \{l', p(l')\}$

to Alice and Bob, where

$$p(l') = \sum_l p(l', l), \quad p(l', l) = p(l'|l)p(l). \quad (3)$$

This process projects the private systems $\{A, B\}$ onto the conditional state

$$\rho_{AB|L'} = \text{Tr}_E(\rho_{ABE|L'}), \quad (4)$$

where

$$\rho_{ABE}(l') \equiv \frac{1}{p(l')} \sum_l p(l', l) \rho_{ABE}(l) = \sum_l p(l|l') \rho_{ABE}(l). \quad (5)$$

Notice that, if L' is completely unrelated to L , then Eve realizes a denial of service, being the communication of the fake variable equivalent to tracing over systems $\{A', B'\}$. In other words, for $p(l', l) = p(l')p(l)$, we have $\rho_{AB|L'} = \rho_A \otimes \rho_B$, where $\rho_A \equiv \text{Tr}_{A'}(\rho_{AA'})$ and $\rho_B \equiv \text{Tr}_{B'}(\rho_{BB'})$.

Secret-key rate: General analysis

After M rounds of the protocol, Alice and Bob will share M copies $(\rho_{AB|L'})^{\otimes M}$. Note that, in general, Alice and Bob do not know anything about the physical process within the UTP, i.e., they do not know the couple $\{\mathbf{T}, L \rightarrow L'\}$. For this reason, what they actually get are M copies of an unknown state $\rho_{AB}^?$ plus classical information L' . However, by measuring a suitable number M' of these copies, they are able to deduce the explicit form of the conditional state $\rho_{AB|L'}$ for the remaining $N = M - M'$ copies (here M, M' and N are large numbers). Then, by applying local measurements, Alice on her private systems and Bob on his, they are able to extract two correlated classical variables, X and Y . Finally, from these variables, they can derive a shared secret key via the classical techniques of error correction (EC) and privacy amplification (PA). These procedures can be implemented using one-way classical communications between these two parties.

Let us bound the secret-key rate of the protocol. For simplicity we omit here the conditioning on L' , so that Eq. (4) simply becomes $\rho_{AB} = \text{Tr}_E(\rho_{ABE})$. It is understood that the final result must be averaged over L' . Independently from its generation, the (generally) mixed state ρ_{AB} can be purified in a pure state $\Phi_{ABe} = |\Phi\rangle\langle\Phi|_{ABe}$ by introducing a suitable system “ e ” to be assigned to Eve (this is generally larger than the E system considered before). After this purification, the scenario is the one depicted in Fig. 2. Here, for every bipartition of the systems, $\{AB, e\}$, $\{Ae, B\}$, or $\{Be, A\}$, the corresponding reduced states have the same von Neumann entropy. In particular, we have $S(\rho_{AB}) = S(\rho_e)$.

Now suppose that Alice performs a POVM $\mathcal{M}_A = \{\hat{A}(x)\}$ on her system A with classical outcome x . This measurement projects Φ_{ABe} onto the conditional state

$$\Phi_{Be}(x) = \frac{1}{p(x)} \text{Tr}_A \left[\hat{A}(x) \Phi_{ABe} \hat{A}(x)^\dagger \right], \quad (6)$$

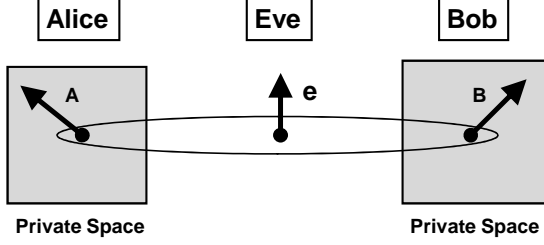


FIG. 2: Purified scenario.

where

$$p(x) = \text{Tr}_{ABe} [\hat{A}(x) \Phi_{ABe} \hat{A}(x)^\dagger]. \quad (7)$$

Thus Alice encodes the stochastic variable $X = \{x, p(x)\}$ in the nonlocal ensemble $\mathcal{E}_{Be} \equiv \{\Phi_{Be}(x), p(x)\}$. Given the conditional state $\Phi_{Be|X}$ of Eq. (6), Bob and Eve can only access their local states, respectively given by

$$\rho_B(x) = \text{Tr}_e [\Phi_{Be}(x)], \quad \rho_e(x) = \text{Tr}_B [\Phi_{Be}(x)]. \quad (8)$$

Thus, on his side, Bob has the ensemble $\mathcal{E}_B \equiv \{\rho_B(x), p(x)\}$, whose measurement estimates Alice's variable X . Assuming that Bob has a quantum memory, he can collect all the private systems B associated to the N rounds of the protocol. Then, asymptotically for $N \rightarrow \infty$, Bob can reach the Holevo bound [17]

$$I(X : B) = S(\rho_B) - \sum_x p(x) S[\rho_B(x)]. \quad (9)$$

At the same time, Eve's information is bounded by

$$I(X : e) = S(\rho_e) - \sum_x p(x) S[\rho_e(x)]. \quad (10)$$

Assuming one-way CCs from Alice to Bob (for implementing EC and PA), we can write the secret-key rate as a difference of Holevo informations [18], i.e.,

$$R = I(X : B) - I(X : e). \quad (11)$$

If we now assume that Alice's POVM is rank one, then the conditional state $\Phi_{Be|X}$ is pure and, therefore, $\rho_{B|X}$ and $\rho_{e|X}$ have the same entropy, i.e., $S[\rho_B(x)] = S[\rho_e(x)]$. As a consequence, we can write

$$R = S(\rho_B) - S(\rho_e) = S(\rho_B) - S(\rho_{AB}) = I(A)B, \quad (12)$$

where $I(A)B$ is the coherent information between Alice and Bob. Thus the secret-key rate is lower-bounded by the entanglement-distillation rate.

Secret-key rate: Detailed analysis

Here we make a more detailed analysis which is more closely connected to the scenario of Fig. 1. In fact, the rate R of Eq. (12) comes from the general configuration

of Fig. 2, which is independent from the actual process generating the final state of Alice and Bob. If we explicitly consider the peculiarities of the scheme of Fig. 1, then we could achieve a larger rate $R^* \geq R$. This new rate can be achieved if Alice and Bob have some knowledge of the classical unreliability of the UTP, i.e., of the amount of information which is "absorbed" by the classical channel $L \rightarrow L'$. Thus, if Eve tries to tamper with the overall security by employing fake CCs, then Alice and Bob can potentially extract a secret-key with rate larger than the entanglement-distillation rate.

In this section, we take the different conditionings (by L and L') explicitly into account. After the CC of $L' = \{l', p(l')\}$, Alice and Bob possess the conditional state $\rho_{AB}(l')$ of Eq. (4). Let us assume that Alice performs a POVM $\mathcal{M}_A = \{\hat{A}(x)\}$ on her system A with classical outcome x . This generates the doubly-conditional state

$$\rho_B(x, l') = \frac{1}{p(x|l')} \text{Tr}_A [\hat{A}(x) \rho_{AB}(l') \hat{A}(x)^\dagger], \quad (13)$$

where

$$p(x|l') = \text{Tr}_{AB} [\hat{A}(x) \rho_{AB}(l') \hat{A}(x)^\dagger]. \quad (14)$$

Averaging over the CCs, the output of Alice's measurement is the unconditional variable $X = \{x, p(x)\}$, where

$$p(x) = \sum_{l'} p(x|l') p(l') = \text{Tr}_A [\hat{A}(x) \rho_A \hat{A}(x)^\dagger]. \quad (15)$$

This is the secret variable to be estimated by Bob. In his private system B , Bob has the ensemble

$$\mathcal{E}_B = \{p(x, l'), \rho_B(x, l')\}, \quad (16)$$

where $p(x, l') = p(x|l') p(l')$. Clearly, this ensemble depends on both X and L' . Exploiting his knowledge of L' , Bob applies a conditional measurement $\mathcal{M}_{B|L'}$ to his system B which estimates the value x encoded by Alice. Asymptotically (i.e., for $N \rightarrow \infty$), using a quantum memory and averaging over the CCs (i.e., over L'), Bob can reach the conditional Holevo information [19]

$$I(X : B|L') = \sum_{l'} p(l') I(X : B|L' = l'). \quad (17)$$

For Eve we have to consider the different conditioning given by L . Thus, the conditional state that Eve shares with Alice is

$$\rho_{AE|L} = \text{Tr}_B (\rho_{ABE|L}), \quad (18)$$

which becomes $\rho_{E|XL}$ after Alice's projection. Explicitly this state is given by

$$\rho_E(x, l) = \frac{1}{p(x|l)} \text{Tr}_A [\hat{A}(x) \rho_{AE}(l) \hat{A}(x)^\dagger], \quad (19)$$

where

$$p(x|l) = \text{Tr}_{AB} [\hat{A}(x)\rho_{AE}(l)\hat{A}(x)^\dagger]. \quad (20)$$

Thus, Eve has the ensemble

$$\mathcal{E}_E = \{p(x, l), \rho_E(x, l)\}, \quad (21)$$

where $p(x, l) = p(x|l)p(l)$. Asymptotically, Eve can eavesdrop $I(X : E|L)$ bits per copy [20].

As a result, we can write the secret-key rate

$$R^* = I(X : B|L') - I(X : E|L'). \quad (22)$$

This quantity can be rewritten as $R^* = R' + \Delta$, where

$$R' \equiv I(X : B|L') - I(X : E|L'), \quad (23)$$

and $\Delta \equiv I(X : E|L') - I(X : E|L)$, quantifies the information which is “absorbed” by the classical channel $L \rightarrow L'$. We call Δ the “classical cheating” by Eve. Clearly, we have $\Delta = 0$ for $L' = L$. R' is the “apparent rate”, which refers to the apparent scenario where Alice, Bob and Eve are all subject to the same conditioning L' . In other words, R' is computed assuming the total state $\rho_{ABE|L'}$, which is then projected onto $\rho_{BE|XL'}$ by Alice’s measurement (see Fig. 3).

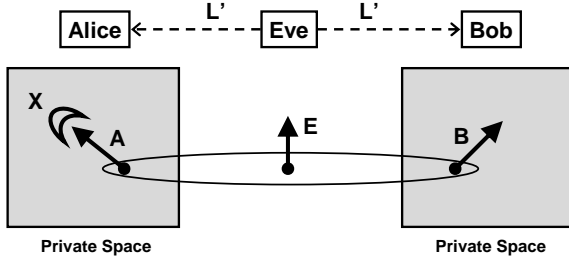


FIG. 3: Conditional state $\rho_{ABE|L'}$ projected onto $\rho_{BE|XL'}$.

We can now easily prove that the secret-key rate is larger than the entanglement-distillation rate. We have the following result (see Supplementary Material for the proof).

Theorem. *Suppose that Eve measures the incoming systems but cheats on the results using a classical channel $L \rightarrow L'$. Then, Alice and Bob’s secret-key rate satisfies*

$$R^* \geq I(A)B|L' + \Delta, \quad (24)$$

where $I(A)B|L'$ is the coherent information conditioned to Eve’s fake variable L' , and Δ is the classical cheating.

Our analysis leaves an intriguing open question. It would be wonderful to provide an explicit example where simultaneously $\Delta > 0$ and $I(A)B|L' = 0$, so that $R^* > 0$. This would imply secret-key distillation without entanglement distillation. More generally, we cannot exclude the possibility that $R^* > I(A)B|L'$ by using POVMs which are not rank one.

Conclusion

We have shown that virtual channels may replace real channels in the QKD setting so as to remove any possibility of side-channel attacks. In its simplest setting, our QKD protocol corresponds to an entanglement swapping experiment, where the dual teleportation channels act as ideal Hilbert space filters to wipe out side-channel attacks. The authenticated users’ private spaces are designed so that any incoming quantum signal is topologically excluded from access to detectors, detector settings or state-generation settings, thus side-channel probing attacks of the private spaces are eliminated. Finally, an external untrusted party performs a suitable LOCC (such as a Bell-state measurement) to create correlations necessary for shared key generation.

Acknowledgments

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Note added

We noticed the manuscript of Lo *et al.* [21] submitted to the arxiv just four days before our submission. At a 2009 Dagstuhl seminar, SLB told the first author of that manuscript the detailed ingredients needed to turn entanglement swapping into a completely side-channel free QKD scheme. Despite this, we stress that there are several important differences between our work and that of Lo *et al.* [21] which itself is based on previous results by Biham *et al.* and Inamori [15].

1) *Generality:* Our work considers a QKD setup involving general quantum systems (e.g., they could be qubit-systems or bosonic modes). In the case of Lo *et al.* [21], the setup is strictly related to the use of decoy states.

2) *Untrusted party:* Our work does not impose any restriction for the action of the middle eavesdropper, who implements a general quantum instrument with classical information communicated to Alice and Bob. Despite this general scenario, we can easily prove that the secret-key rate is lowerbounded by the coherent information. In Lo *et al.* [21], a very strong conjecture is made explicitly for the middle eavesdropper, who is always assumed to perform a Bell measurement. Thus, in practice, the third party cannot be regarded as an eavesdropper (Eve) but more correctly as a trusted party (Charlie). The real action of Eve is therefore restricted to the noise present in the quantum channels between the honest users and Charlie.

3) *Trojan-horses:* Our work excludes the possibility that the eavesdropper is able to get information by sending trojan-horses into Alice’s and Bob’s apparatus. This is the result of a Hilbert space filtering which derives from

a suitable use of the entanglement in Alice’s and Bob’s private spaces (this aspect is further discussed in our Supplementary Material). By contrast, in the paper Lo *et al.* [21], state preparation can be easily eavesdropped upon, being directly inline with the output ports of Alice’s and Bob’s private spaces.

Finally, in Lo *et al.* [21] there is a disconnect between the security proof (which relies on Alice and Bob creating entanglement) and the implementation. We note however that the authors cite Ref. [22] as a possible way to avoid quantum information processing but do not seriously discuss this as part of their proof in any detail.

Supplementary Material

IN DEFENSE OF PRIVATE SPACES

In quantum cryptography unconditional security proofs are derived under the assumption that Alice’s and Bob’s apparatus (private spaces) are completely inaccessible by an eavesdropper who, therefore, can only attack the signal systems which are transmitted through the quantum communication channel connecting the two parties. Under this assumption, secret-key rates and security thresholds are derived in both discrete and continuous variable quantum key distribution.

One potential loophole in the security proofs is related to how a theoretical protocol is actually implemented experimentally. Any redundant information encoded in extra degrees of freedom or extra Hilbert space dimensions outside the theoretical prescription can allow for so-called side-channel attacks. By their nature, such attacks may be of classical or quantum degrees of freedom and are insidious because even quantifying their threat appears to involve understanding what have been called unknown unknowns about the vulnerability of the experimental set-up.

Progress has been made on eliminating side-channel attacks in the quantum communication channels between private spaces, but this leaves open potential attacks on the private spaces through their quantum communication ports. Let us therefore take a step back and consider private spaces in more detail: What goes on in Alice’s and Bob’s private spaces involves a significant amount of classical information processing; at the very least the key itself will be generated and stored as classical information. Now with virtually any technology we have today classical information is stored, processed and transmitted in a highly redundant fashion (many electrons are used to charge a capacitor to represent a bit value, or many electrons must pass through the base junction of a transistor to effect a logical switching operation, tapping on a keyboard produces sound waves and electromagnetic

signals in addition to the ‘legitimate’ electrical signals in the wires, etc). In principle any of this redundant information may leak out of the private space through a “parasite” channel. An eavesdropper might therefore ignore the quantum communication channel and directly attack Alice’s and Bob’s apparatus by exploiting the presence of parasite channels: this is also a “side-channel attack”.

The implicit assumption in quantum cryptography is that we could always improve technology in such a way that Alice’s and Bob’s private spaces are not affected by the presence of parasite channels, so that the legitimate participants do indeed have access to absolutely private spaces. (For instance, Alice and Bob could simulate the classical information processing on a quantum computer. A hacked operating system on such a machine could be tested for by randomly running subroutines that confirm that coherence is preserved and that no information is copied out to where it can be stored or transmitted by a trojan program — see also Ref. [23].)

However, even if you rely on a perfect isolation technology, there remains a potential chink in this armor, which is the quantum communication port used either to transmit a quantum state out of your private space or to accept a quantum state for detection into it.

If you open a communication port for quantum states to enter or leave you must explicitly deal with side channels which can be probing these links to your private space. Eve can potentially send trojan systems through Alice’s and Bob’s communication ports and detect their reflection to infer both state preparation and measurement settings. As an example, in the standard BB84 protocol, Eve can irradiate Alice’s apparatus by using optical modes at slightly different frequencies. Then, from reflection, Eve can infer the polarization chosen in each round of the protocol. Thanks to this information, Eve can measure each signal system in the correct basis. Another example regards the so-called plug-and-play systems, where trojan systems can be reflected together with signal systems, as discussed in Ref. [10].

Our paper shows how to overcome the problem of the open quantum communication ports, therefore making feasible the notion of absolutely private spaces. Note that this problem is not addressed by current device-independent quantum cryptography, where such attacks on the private space ports are simply considered illegitimate as they violate the strong private space assumption. The key point of our scheme is that detectors are no longer “in line” with the quantum communication port of the private space. For this reason, it is not possible for an external party to probe the port and obtain detector settings or readouts from the processing of parasite systems. In order to explain this key feature in detail, we analyze the problem of the quantum communication ports by comparing standard protocols with our scheme.

In Fig. 4, we depict a general prepare-and-measure protocol, where Alice’s variable X is encoded in a quantum

state $\rho(X)$ by modulation. Bob's variable Y is the output of a quantum measurement. Here, Eve can attack the quantum communication ports by using two trojan systems e and f . By means of e , Eve can retrieve information about the state preparation $X \rightarrow \rho(X)$. By means of f , she can retrieve information about the measurement apparatus of Bob and, therefore, about Y .

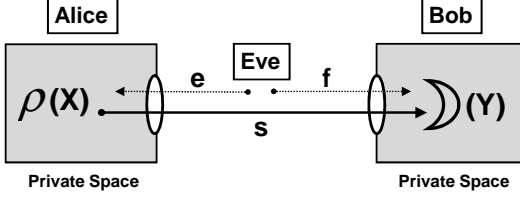


FIG. 4: Port attack in a prepare and measure protocol.

In Fig. 5, we depict a general entanglement-based protocol, where an untrusted party (Eve) distributes entanglement between two parties. This is done by distributing an entangled state $\rho = \rho_{AB}$, where system A is sent to Alice and system B is sent to Bob. Alice and Bob can perform entanglement distillation and measure the output distilled systems to derive two correlated classical variables, X and Y , respectively. In this scenario, Eve can decide not to attack the source ρ but directly the two quantum communication ports of Alice and Bob. Eve can probe these ports by using two trojan systems e and f , which can retrieve information about Alice's and Bob's distilling and detecting apparatus. As a result, Eve can infer information about X and Y .

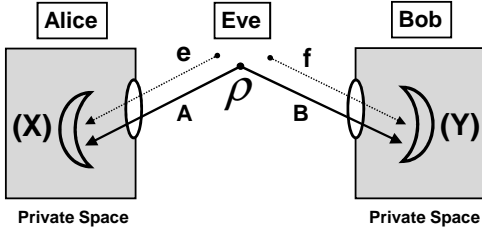


FIG. 5: Port attack in an entanglement-based protocol.

In Fig. 6, we depict our protocol where an untrusted party (Eve) represents an entanglement swapper between Alice and Bob. This is generally done by measuring two *public* systems, A' and B' , received from Alice and Bob,

processing the outcome of the measurement, and classically communicating the processed data back to Alice and Bob. As a result the two private systems, A and B , become correlated, so that Alice and Bob can extract two correlated classical variables, X and Y , by applying suitable measurements. In particular, if Alice and Bob can access quantum memories, then they can extract a secret key at a rate which is at least equal to the coherent information between A and B . Eve can attempt a side-channel attack against the two ports by sending two trojan systems e and f . In this case, however, the apparatus which detect the two private systems A and B are inaccessible to Eve. By exploiting reflections from the ports, Eve can only retrieve information regarding the reduced states $\rho_{A'}$ and $\rho_{B'}$ of the two public systems A' and B' . However, these reduced states contain no useful information about the private system A or B or Alice's or Bob's detector settings or outputs.

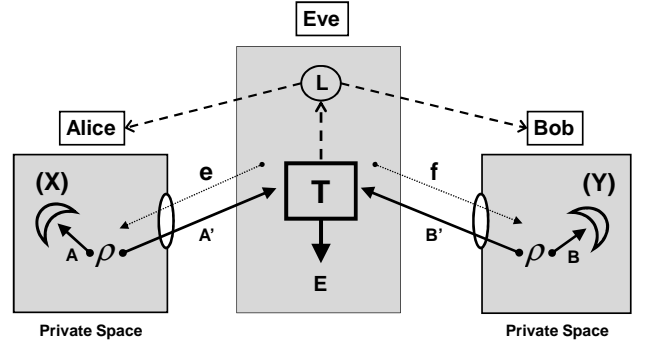


FIG. 6: Port attack in our scheme.

To understand better how the full isolation of the private systems might be achieved, we may consider the procedure depicted in Fig. 7. It is explained for Alice's private space, but steps are identical for Bob.

In the first step (a), Alice's port is closed and she prepares an entangled state $\rho = \rho_{AA'}$ where system A is directed towards a quantum memory (QM), while system A' is directed towards a delay line (DL). In step (b), once system A is stored in the memory and while system A' is trapped in the delay line, a shutter is used to fully separate the delay line from the rest of Alice's apparatus. Note that a virtual channel between A and A' has been created. In step (c), Alice's quantum communication port is opened and system A' is transmitted to Eve. During this stage, trojan systems may enter the port but no detector is in line with the port. In step (d), the port is closed with the private system A kept in the memory. The previous steps (a)-(d) are repeated many times, so that Alice collects many private systems in her quantum memory. We therefore reach step (e) of the figure. Finally, once Alice has received all the classical communications, she applies a collective quantum measurement on her quantum memory to retrieve the classical variable

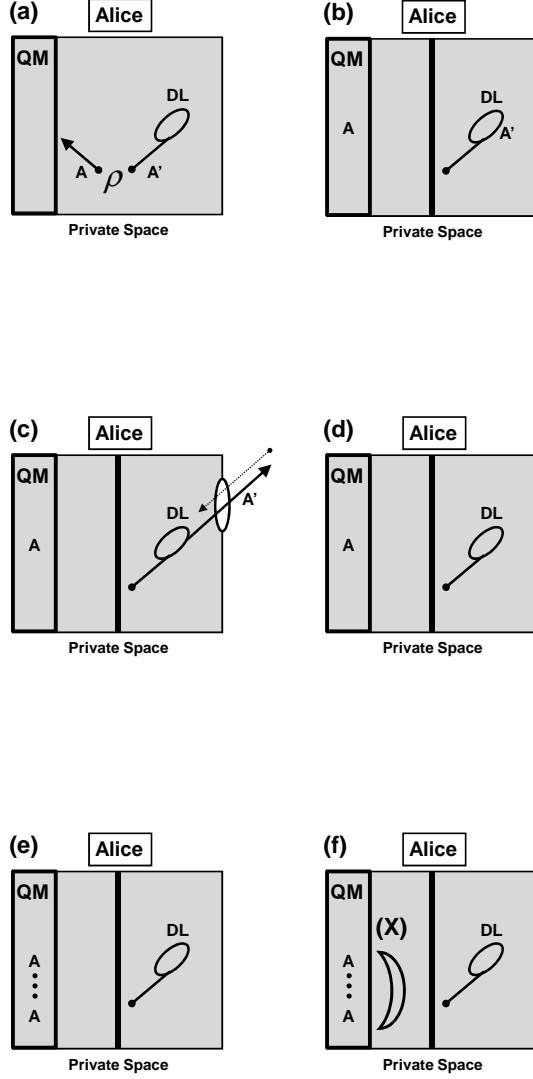


FIG. 7: Possible procedure for the full isolation of the private systems. See text for explanations.

X . This measurement can include or be anticipated by an entanglement distillation.

NOTATION AND BASIC FORMULAS

In part of the derivation we adopt the enlarged Hilbert space (EHS) representation, where stochastic classical variables are embedded in quantum systems. Consider a stochastic variable $X = \{x, p(x)\}$ which is encoded into an ensemble of states of some quantum system A , i.e.,

$$\mathcal{E}_A = \{p(x), \rho_A(x)\}. \quad (25)$$

This ensemble may be equivalently represented by the classical-quantum (CQ) state

$$\rho_{\mathbf{X}A} = \sum_x p(x) |x\rangle \langle x|_{\mathbf{X}} \otimes \rho_A(x), \quad (26)$$

where the stochastic variable X is embedded into the dummy quantum system \mathbf{X} , by using an orthonormal basis $\{|x\rangle\}$ in the Hilbert space $\mathcal{H}_{\mathbf{X}}$ of \mathbf{X} . We denote by $\rho_A(x)$ the state of a system A which is conditioned by the value x of a stochastic variable X . The notation $\rho_{A|X}$ refers to the conditional state $\rho_A(x)$ where x is not specified. Clearly, we have

$$\rho_A = \sum_x p(x) \rho_A(x). \quad (27)$$

Given a quantum system A in a state ρ_A , its von Neumann entropy $S(\rho_A)$ is also denoted by $H(A)$. Given a quantum system \mathbf{X} , embedding the stochastic variable X , its quantum entropy $H(\mathbf{X})$ is just the Shannon entropy $H(X)$. Given two quantum systems, A and B , we denote by $I(A : B)$ their quantum mutual information. This is defined by

$$I(A : B) = H(B) - H(B|A), \quad (28)$$

where $H(B|A) = H(AB) - H(A)$ is the conditional quantum entropy. Note that $H(B|A)$ can be negative and it is related to the coherent information by the relation

$$I(A)B = -H(B|A). \quad (29)$$

For $A = \mathbf{X}$, the quantum mutual information $I(A : \mathbf{X})$, which is computed over the CQ-state of Eq. (26), corresponds to the Holevo information $I(A : X)$, computed over the ensemble of Eq. (25). For $A = \mathbf{X}$ and $B = \mathbf{Y}$, embedding two stochastic variables X and Y , $I(\mathbf{X} : \mathbf{Y})$ is just the classical mutual information $I(X : Y)$. For three quantum systems A , B , and C , we can consider the conditional quantum mutual information

$$I(A : B|C) = H(AC) + H(BC) - H(ABC) - H(C), \quad (30)$$

which is ≥ 0 as a consequence of the strong subadditivity of the von Neumann entropy. For a classically correlated system $C = \mathbf{X}$, we have a probabilistic average over mutual informations, i.e.,

$$I(A : B|\mathbf{X}) = I(A : B|X) \equiv \sum_x p(x) I(A : B|X = x). \quad (31)$$

List of other useful elements:

- Given a tripartite quantum system ABC , we can use the “chain rule”

$$I(A : BC) = I(A : B) + I(A : C|B). \quad (32)$$

- Invariance of the Holevo information under addition of classical channels, i.e., for a classical channel

$$p(y|x) : X \rightarrow Y, \quad (33)$$

we have

$$I(A : X) = I(A : XY). \quad (34)$$

- Given a Markov chain $X \rightarrow Y \rightarrow Z$, the classical mutual information decreases under conditioning [24], i.e.,

$$I(X : Y|Z) \leq I(X : Y). \quad (35)$$

Notice that, for three general stochastic variables, we have $I(X : Y|Z) \geq I(X : Y)$, so that the so-called “interaction information”

$$I(X : Y : Z) \equiv I(X : Y|Z) - I(X : Y), \quad (36)$$

can be positive, negative or zero.

- Data processing inequality. For a Markov chain $X \rightarrow Y \rightarrow Z$, we have

$$H(X) \geq I(X : Y) \geq I(X : Z). \quad (37)$$

PROOF OF THE THEOREM

Let us purify the mixed state $\rho_{ABE|L'}$ into the pure state $\Phi_{ABE\tilde{E}|L'} = |\Phi\rangle\langle\Phi|_{ABE\tilde{E}|L'}$ by introducing an ancillary system \tilde{E} which is assumed to be in Eve’s hands (so that Eve’s global system consists of $E\tilde{E}$). This scenario is depicted in Fig. 8.

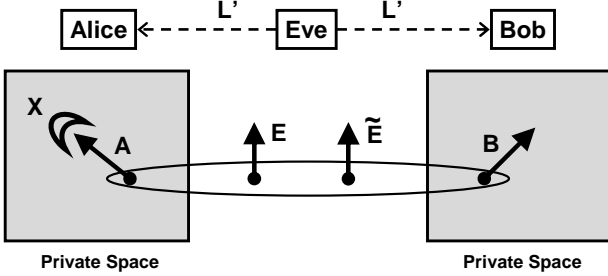


FIG. 8: Purification. Conditional state $\Phi_{ABE\tilde{E}|L'}$ projected onto $\Phi_{BE\tilde{E}|XL'}$.

Thus, for the total state $\rho_{ABE|L'}$, we have

$$\rho_{ABE}(l') = \text{Tr}_{\tilde{E}} [\Phi_{ABE\tilde{E}}(l')]. \quad (38)$$

For the conditional state $\rho_{BE|XL'}$, generated by the measurement, we can write

$$\begin{aligned} \rho_{BE}(x, l') &= \frac{1}{p(x|l')} \text{Tr}_A [\hat{A}(x) \rho_{ABE}(l') \hat{A}(x)^\dagger] \\ &= \frac{1}{p(x|l')} \text{Tr}_{A\tilde{E}} [\hat{A}(x) \Phi_{ABE\tilde{E}}(l') \hat{A}(x)^\dagger] \\ &= \text{Tr}_{\tilde{E}} [\Phi_{BE\tilde{E}}(x, l')], \end{aligned} \quad (39)$$

where

$$\Phi_{BE\tilde{E}}(x, l') \equiv \frac{1}{p(x|l')} \text{Tr}_A [\hat{A}(x) \Phi_{ABE\tilde{E}}(l') \hat{A}(x)^\dagger], \quad (40)$$

represents the conditional state $\Phi_{BE\tilde{E}|XL'}$, which is generated by the measurement in the purified scenario. Clearly if we discard X , we get the reduced state

$$\Phi_{BE\tilde{E}|L'} \equiv \langle \Phi_{BE\tilde{E}|XL'} \rangle_X = \text{Tr}_A [\Phi_{ABE\tilde{E}|L'}]. \quad (41)$$

Because of Eq. (39), the conditional state $\Phi_{BE\tilde{E}|XL'}$ can be used to compute R' via

$$\begin{aligned} R' &\equiv I(X : B|L')_\rho - I(X : E|L')_\rho \\ &= I(X : B|L')_\Phi - I(X : E|L')_\Phi, \end{aligned} \quad (42)$$

where $\rho = \rho_{BE|XL'}$ and $\Phi = \Phi_{BE\tilde{E}|XL'}$ (the computation is exactly the same up to a trace over \tilde{E}). In the EHS representation, the conditional state $\Phi_{BE\tilde{E}|XL'}$ becomes

$$\Psi_{\mathbf{X}L'|BE\tilde{E}} = \sum_{x, l'} p(x, l') |x\rangle\langle x|_{\mathbf{X}} \otimes |l'\rangle\langle l'|_{L'} \otimes \Phi_{BE\tilde{E}}(x, l'). \quad (43)$$

Thus, we can also set

$$R' = I(\mathbf{X} : B|L')_\Psi - I(\mathbf{X} : E|L')_\Psi, \quad (44)$$

where $\Psi = \Psi_{\mathbf{X}L'|BE\tilde{E}}$. From the chain rule we have

$$\begin{aligned} I(\mathbf{X} : E\tilde{E}|L')_\Psi &= I(\mathbf{X} : E|L')_\Psi + I(\mathbf{X} : \tilde{E}|EL')_\Psi \\ &= I(\mathbf{X} : E|L')_\Psi + \gamma, \end{aligned} \quad (45)$$

where $\gamma \equiv I(\mathbf{X} : \tilde{E}|EL')_\Psi \geq 0$ is the information contribution due to the purification [25]. In other words, the (conditional) Holevo information can only increase with the purification, i.e.,

$$I(X : E\tilde{E}|L') = I(X : E|L') + \gamma \geq I(X : E|L'). \quad (46)$$

As a consequence, we have $R' = R'' + \gamma$, where

$$R'' \equiv I(X : B|L')_\Phi - I(X : E\tilde{E}|L')_\Phi. \quad (47)$$

In terms of conditional entropies, we have

$$\begin{aligned} R'' &= H(B|L')_\Phi - H(B|XL')_\Phi \\ &\quad - [H(E\tilde{E}|L')_\Phi - H(E\tilde{E}|XL')_\Phi]. \end{aligned} \quad (48)$$

Here $H(E\tilde{E}|L')$ is computed over $\Phi = \Phi_{BE\tilde{E}|XL'}$, discarding X and B , i.e., over the reduced state

$$\Phi_{EE|L'} = \text{Tr}_{AB} [\Phi_{ABE\tilde{E}|L'}]. \quad (49)$$

Now since $\Phi_{ABE\tilde{E}|L'}$ is pure, we have $H(E\tilde{E}|L') = H(AB|L')$, where $H(AB|L')$ can be computed over $\rho_{AB|L'} = \text{Tr}_{E\tilde{E}}[\Phi_{ABE\tilde{E}|L'}]$. Clearly, also $H(B|L')_\Phi$ can

be computed over $\rho_{AB|L'}$. As a consequence we can recognize in Eq. (48) the conditional coherent information

$$I(A)B|L' = H(B|L') - H(AB|L'),$$

associated with Alice and Bob's conditional state $\rho_{AB|L'}$. Thus, we can set

$$R'' = I(A)B|L' + [H(E\tilde{E}|XL')_{\Phi} - H(B|XL')_{\Phi}]. \quad (50)$$

Here, we can assume that Alice's measurement is a rank one POVM. As a result, $\Phi = \Phi_{BE\tilde{E}|XL'}$ is also a pure state, and we can set $H(E\tilde{E}|XL')_{\Phi} = H(B|XL')_{\Phi}$, so that $R'' = I(A)B|L'$. Finally, we can write

$$\begin{aligned} R^* &= R'' + \gamma + \Delta \\ &= I(A)B|L' + \gamma + \Delta \\ &\geq I(A)B|L' + \Delta, \end{aligned} \quad (51)$$

where we have used $\gamma \geq 0$ from its definition.

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- [19] Equivalently, we can adopt the EHS representation (see Supplementary Material for details), where the ensemble \mathcal{E}_B and the stochastic variables X and L' are described by a unique classical-quantum state $\rho_{\mathbf{X}\mathbf{L}'B} = \sum_{x,l'} p(x,l') |x\rangle \langle x|_{\mathbf{X}} \otimes |l'\rangle \langle l'|_{\mathbf{L}'} \otimes \rho_B(x,l)$. The Holevo quantity of Eq. (17) corresponds to the conditional quantum mutual entropy $I(\mathbf{X} : B|\mathbf{L}')$ computed over this state.
- [20] Equivalently, we can consider the classical-quantum state $\rho_{\mathbf{X}\mathbf{L}E} = \sum_{x,l} p(x,l) |x\rangle \langle x|_{\mathbf{X}} \otimes |l\rangle \langle l|_{\mathbf{L}} \otimes \rho_E(x,l)$, and compute $I(\mathbf{X} : E|\mathbf{L}) = I(X : E|L)$.
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